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Goldstone Bosons in the Appelquist-Terning ETC Model

Bhashyam Balaji

Physics Department
Boston University
Boston, MA 02215

Abstract

It is demonstrated that the extended technicolor model proposed recently by Appelquist and Terning has pair of potentially light $U(1)$ Goldstone bosons coupling to ordinary matter with strength $\frac{2m_f}{F_\pi}$, where m_f is the mass of the fermion and $F_\pi \approx 125\,\mathrm{GeV}$. These Goldstone bosons could get a mass if the spontaneously broken $U(1)$ symmetries are also explicitly broken, by physics beyond that specified in the model. An attempt to break these symmetries by embedding the model into a larger gauge group seems to be inadequate. The problem is because there are too many representations and there is a mismatch between the number of condensates and the number of gauge symmetries broken.

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1. INTRODUCTION

The reasons for considering the standard model of electroweak interactions to be incomplete are well known. In particular, the scalar Higgs sector possesses unsatisfactory features such as the naturalness/gauge hierarchy problem [1], arbitrariness of Yukawa couplings and triviality [2]. These are particularly disturbing since electroweak symmetry breaking is responsible for endowing ordinary fermions and the weak gauge bosons with their masses.

One proposal for eliminating these problems assumes that there are no fundamental Higgs scalars. Instead, one postulates the existence of a new set of fermions—technifermions—which interact via a new parity-conserving strong force called technicolor [3]. Crudely speaking, 'standard' technicolor mimics QCD at a higher energy scale. Walking technicolor [4] modifies relations obtained by naive scaling from QCD, due to large anomalous dimensions of relevant composite operators. Technicolor enables us to solve part of the problem —gauge boson mass generation—based on nontrivial dynamics.

The dynamical generation of fermion masses requires yet another interaction — Extended Technicolor (ETC) [5] [6]. It generates the current algebra masses of ordinary fermions by communicating the dynamical technifermion masses to the ordinary fermions. In other words, ETC gauge bosons couple ordinary fermions to technifermions, and couple to the various flavors differently.

Three of the Goldstone bosons produced by the spontaneous chiral symmetry breaking of techniflavor symmetry contribute to $W^{\pm}$ and Z^0 masses. 'Realistic' models usually contain other (pseudo-) Goldstone bosons, most of which acquire some mass from color or electroweak interactions [6].

An ETC model explaining the wide range of values of ordinary fermion masses and the CKM matrix elements has been elusive. In particular, it is hard to construct models compatible with experiments. Strong constraints from flavor-changing neutral current experiments ruled out QCD-like models long ago. Moreover, there are unwanted massless goldstone bosons in any extended technicolor model with too many fermion representations [6].

Recently, severe constraints have also come from precision electroweak measurements. Assuming the scale of new physics to be large compared to the W mass, it is found that there are important corrections to electroweak observables that are 'oblique'—i.e., correction to gauge boson propagators [7]. These oblique corrections are encoded in three

parameters—S, T and U [8]. The parameter T [9] is a measure of weak-isospin breaking—smallness of T means that technicolor models with large weak isospin breaking (needed to generate the b-t mass difference, for instance) are severely constrained. Calculations of the S parameter in QCD-like technicolor models with isospin symmetry indicates that technicolor models with too many representations (the one-family technicolor model, for instance) are incompatible with the experimental value of S. Of course, those models are already ruled out on other considerations [6]. However, it is unclear if walking technicolor models are incompatible with the experimental values of S and T [10] or corrections to the $Zb\bar{b}$ vertex [11] , due to the difficulties inherent in performing reliable calculations in a strongly-interacting theory.

An attempt in this direction was made recently by Appelquist and Terning [12]. They constructed an ETC model and used it to produce a wide range of fermion masses. They also argued how the model could be compatible with experimental constraints such as the value of S, FCNC and small neutrino masses.

In this paper, we will begin by demonstrating the existence of two potentially light U(1) Goldstone bosons in the Appelquist-Terning model. Appelquist and Terning only specify gauge interactions below 1000 TeV and this is what we take to be the model. They also discuss the need for non-renormalizable operators arising from physics beyond 1000 TeV. Those operators could give mass to the two goldstone bosons by explicitly breaking their chiral symmetries. We shall discuss such a possibility assuming that these non-renormalizable operators arise from a gauge theory. It will be shown that simple extensions, such as embedding into larger groups, will not solve the problem. Our analysis will naturally lead us to the main reason for the problem—a mismatch between the number of broken diagonal gauge generators and the number of condensates. Note that the U(1) Goldstone bosons discussed here are different from the 'P^0' discussed by Eichten and Lane [6]—those are avoided by implementing Pati-Salam unification and avoiding repeated representations.

For brevity, the potentially light pseudo-Goldstone bosons will be called axions. However, they have nothing to do with the conventional Peccei-Quinn axion since they couple to anomaly free currents. In Section 2, we briefly describe the model, paying attention to the symmetry breaking pattern. In Section 3, we show the existence of the unwanted massless Goldstone bosons and present an analysis of the general reasons for the problem. We discuss the inadequacy of some 'natural' ways of addressing the problem and present a solution. Finally, we present our conclusion.

2. The Appelquist—Terning Model

The gauge group is taken to be $SU(5)_{ETC} \otimes SU(2)_{HC} \otimes SU(4)_{PS} \otimes SU(2)_L \otimes U(1)_R$ with fermion content (all fermions taken to be left-handed):

$$
\begin{array}{ll}
\psi_1 = (5,1,4,2)_0 & \psi_5 = (1,2,6,1)_0 \\
\psi_2 = (\overline{5},1,\overline{4},1)_1 & \psi_6 = (10,1,1,1)_0 \\
\psi_3 = (\overline{5},1,\overline{4},1)_{-1} & \psi_7 = (5,1,1,1)_0 \\
\psi_4 = (1,1,6,1)_0 & \psi_8 = (\overline{10},2,1,1)_0
\end{array}
\tag{1}
$$

Here $SU(2)_{HC}$ is an additional strong gauge group which is needed to help break the $SU(5)_{ETC}$ down to $SU(2)_{TC}$. Hypercharge, Y,(normalized by $Q = T_{3L} + Y/2$) is given by $Y = Q_R + T_{15}^{PS}$, where $T_{15}^{PS} = \mathrm{diag}(1/3, 1/3, 1/3, -1)$ is a generator of the $SU(4)_{PS}$ Pati-Salam group which implements quark-lepton unification (For details regarding the motivation for the choice of the gauge group and fermion representation content see [12]).

At the Pati-Salam breaking scale (taken to be around 1000 TeV), a condensate is assumed to form in the channel $(\overline{5},1,\overline{4},1)_{-1} \times (5,1,1,1)_0 \to (1,1,\overline{4},1)_{-1}$. This channel ($\langle\psi_3\psi_7\rangle \neq 0$) is not the most attractive Channel (MAC) [13]. Instead, new physics is presumed to trigger its formation. This condensate breaks the $U(1)_R$ and $SU(4)_{PS}$ gauge groups leading to the gauge group $SU(5)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ below Λ_{PS}.

Next, it is assumed that at $\Lambda_5 \approx 1000\,\mathrm{TeV}$ a condensate forms in the channel $(10,1,1,1)_0 \times (10,1,1,1)_0 \to (\overline{5},1,1,1)_0$ —i.e., $\langle\psi_6\psi_6\rangle \neq 0$. The singlet channel $10 \times \overline{10} \to 1$ is disfavored as $SU(2)_{HC}$ is assumed to be relatively strong, so as to resist breaking. Then, the condensate $(\overline{5},1,1,1)_0$ breaks the gauge symmetry to $SU(4)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

The next condensation is the attractive channel $(\overline{4},2,1,1)_0 \times (6,2,1,1)_0 \to (4,1,1,1)$ — $\langle\psi_8\psi_8\rangle_1 \neq 0$ — which is taken to occur at $\Lambda_4 \approx 100\,\mathrm{TeV}$. (The condensate is subscripted to distinguish this channel from another at Λ_3 arising from a different piece in ψ_8.) The so-called 'big MAC' criterion is applied here. When two or more relatively strong gauge interactions are at play, the favored breaking channel is determined by the sum of the interactions. It is a generalization of the ordinary MAC criterion. Hence, below Λ_4 the gauge group is $SU(3)_{ETC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

The final stage of ETC breaking takes place at the scale $\Lambda_3 \approx 10\,\mathrm{TeV}$ with the big MAC condensate $(\overline{3},2,1,1)_0 \times (\overline{3},2,1,1)_0 \to (3,1,1,1)_0$ — $\langle\psi_8\psi_8\rangle_2 \neq 0$. This breaks

$SU(3)_{ETC}$ to $SU(2)_{TC}$ so that the gauge group below Λ_3 is $SU(2)_{TC} \otimes SU(2)_{HC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Hypercolored paricles are confined at $\Lambda_{HC} \approx \Lambda_3$, and the HC sector decouples from ordinary fermions and technifermions. We then have a one-family technicolor model (actually there is an additional "vector" quark [12]).

Finally, at the technicolor scale Λ_{TC}, the $SU(2)_{TC}$ becomes strong resulting in condensation in the $2 \times 2 \to 1$ channel— $\langle \psi_1 \psi_2 \ \rangle \neq 0$, $\langle \psi_1 \psi_3 \rangle \neq 0$ and $\langle \psi_1 \psi_6 \rangle \neq 0$. This breaks the electroweak gauge group $SU(2)_L \otimes U(1)_Y$ to $U(1)_{\rm em}$.

3. The Axion Analysis

For each fermion representation, there is a global $U(1)$ symmetry current. In general, these currents have gauge anomalies. From these currents, one can form anomaly-free combinations; these correspond to exact global symmetries of the theory. One then follows this global symmetry through the various gauge symmetry breakings and investigates whether or not it is spontaneously broken at any scale. At each stage of symmetry breaking, one forms linear combinations of the gauged and global currents that leave the condensates invariant. These remaining symmetries generate unbroken global $U(1)$'s. The remaining orthogonal combinations couple to the massive gauge bosons [14].

To begin with, there are eight gauge singlet global $U(1)$ currents $j^A_\mu = \overline{\psi_A} \gamma_\mu \psi_A$, where $A = 1, ...8$ corresponding to the eight representations. Each of them has an anomalous divergence due to the strong ETC, PS and/or HC interactions.

$$\begin{aligned}
D_1 &= 8S_5 + 10S_4 \\
D_2 &= 4S_5 + 5S_4 \\
D_3 &= 4S_5 + 5S_4 \\
D_4 &= 2S_4 \\
D_5 &= 6S_2 + 4S_4 \\
D_6 &= 3S_5 \\
D_7 &= S_5 \\
D_8 &= 6S_5 + 10S_2
\end{aligned} \tag{2}$$

Here $D_A = \partial_\mu j^\mu_A$ and $S_n = \frac{g_n^2}{32\pi^2} F_n \cdot \tilde{F_n}$, where $n = 2, 4, 5$ correspond to to gauge groups $SU(2)_{HC}$, $SU(4)_{PS}$ and $SU(5)_{ETC}$ respectively. The electroweak $SU(2)_L$ instanton has a negligible effect around Λ_{PS}, and weak anomalies are ignored.

From these currents, one can form five gauge anomaly free symmetry currents, which are

$$
\begin{aligned}
J_\mu^1 &= j_\mu^1 - j_\mu^2 - j_\mu^3 \\
J_\mu^2 &= j_\mu^6 - 3j_\mu^7 \\
J_\mu^3 &= j_\mu^1 - 5j_\mu^4 - 8j_\mu^7 \\
J_\mu^4 &= -10j_\mu^4 + 5j_\mu^5 + 6j_\mu^6 - 3j_\mu^8 \\
J_\mu^5 &= -j_\mu^2 + j_\mu^3
\end{aligned}
\tag{3}
$$

One of these currents, J_μ^5, is actually the $U(1)_R$ gauge current. So the global symmetry above Λ_{PS} is $U(1)^4$. (To include the $SU(2)_L$ we take three of the four linearly independent current combinations not containing j_μ^1. The symmetry would be $U(1)^3$; this will not change our conclusions. Note that the $U(1)_R$ anomaly is irrelevant for our considerations due to the absence of instantons.) We follow this global symmetry through all of the gauge symmetry breakings; spontaneous breakdown of any global symmetry would result in a corresponding Goldstone boson at that scale.

Below Λ_{PS} one can form four combinations from the above five currents which leave invariant the gauge symmetry breaking condensate $\langle\psi_3\psi_7\rangle \neq 0$—the fifth one being the $U(1)_R$ gauge current [5]. So there still exists a $U(1)^4$ symmetry generated by (keeping the same symbol,J, for these currents)

$$
\begin{aligned}
J_\mu^1 &= -3j_\mu^1 + 3j_\mu^2 + 3j_\mu^3 + j_\mu^6 - 3j_\mu^7 \\
J_\mu^2 &= -7j_\mu^1 + 8j_\mu^2 + 8j_\mu^3 - 5j^4\mu - 8j_\mu^7 \\
J_\mu^3 &= j_\mu^1 - 2j_\mu^2 \\
J_\mu^4 &= -10j_\mu^4 + 5j_\mu^5 + 6j_\mu^6 - 3j_\mu^8
\end{aligned}
\tag{4}
$$

One can proceed to find the four global symmetry currents, which remain conserved below Λ_5, to be

$$
\begin{aligned}
J_\mu^1 &= 18(j_\mu^1 - j_\mu^2 - j_\mu^3) - 10j_\mu^4 + 5j_\mu^5 + 18j_\mu^7 - 3j_\mu^8 \\
J_\mu^2 &= -10j_\mu^4 + 5j_\mu^5 + 6j_\mu^6 - 3j_\mu^8 - 6J_\mu^{5E} \\
J_\mu^3 &= j_\mu^1 - 2j_\mu^2 \\
J_\mu^4 &= -7j_\mu^1 + 8j_\mu^2 + 8j_\mu^3 - 5j_\mu^4 - 8j_\mu^7
\end{aligned}
\tag{5}
$$

Here J_μ^{5E} is the gauge current corresponding to the diagonal generator in SU(5) which is not in SU(4). Note that J_μ^2 is left unbroken although the gauge and global currents

are separately broken. This is analogous to the situation in the standard model where $SU(2)_L \otimes SU(2)_R \to SU(2)_V$. The $SU(5)_E$ generator in the fundamental representation is chosen to be $\mathrm{diag}\frac{1}{2}(-4,1,1,1)$ —this automatically fixes the U(1) charges in the other non- fundamental representations. This is merely a convenient choice of a basis.

One can likewise obtain the global symmetry currents below Λ_4 and Λ_3. Finally, one investigates the $U(1)$ global symmetries below the electroweak symmetry breaking scale. This entails three condensates — $\langle\psi_1\psi_2\rangle \neq 0$, $\langle\psi_1\psi_3\rangle \neq 0$ and $\langle\psi_1\psi_6\rangle \neq 0$. Only two of the four global symmetries are realized in the Wigner-Weyl mode, namely those corresponding to

$$\begin{aligned} J^1_\mu = &-36(j^1_\mu - j^2_\mu - j^3_\mu) + 10j^4_\mu - 25j^5_\mu - 18j^6_\mu \\ &- 36j^7_\mu + 15j^8_\mu + 8J^{5E}_\mu - 24J^{4E}_\mu + 6J^{3E}_\mu \\ \\ J^2_\mu = &-18j^1_\mu + 18j^2_\mu + 20j^3_\mu - 15j^4_\mu - (5/3)j^5_\mu + 2j^6_\mu \\ &- 20j^7_\mu + j^8_\mu - 2J^{5E}_\mu + 2J^{3E}_\mu + J^{SU(2)_3}_\mu \end{aligned} \tag{8}$$

The $SU(2)_3$ generator in the fundamental representation is normalized as $\mathrm{diag}(1,-1)$. The $SU(4)$ and $SU(3)$ generators in the fundamental representation are normalized as $\mathrm{diag}(-3,1,1,1)$ and $\mathrm{diag}(-2,1,1)$ respectively. Note that the j^{em}_μ piece is irrelevant for our purposes since $U(1)_{\mathrm{em}}$ is unbroken.

The other two broken global $U(1)$ currents (up to unbroken pieces) are

$$\begin{aligned} J^3_\mu = &-36(j^1_\mu - j^2_\mu - j^3_\mu) - 30j^4_\mu - 5j^5_\mu + 6j^6_\mu \\ &- 36j^7_\mu + 3j^8_\mu - 6J^{5E}_\mu + 6J^{3E}_\mu \\ \\ J^4_\mu = &-70j^1_\mu + 74j^2_\mu + 72j^3_\mu - 55j^4_\mu + (20/3)j^5_\mu + 8j^6_\mu \\ &- 72j^7_\mu + 4j^8_\mu - 8J^{5E}_\mu + 8J^{3E}_\mu \end{aligned} \tag{9}$$

One can incorporate the effect of the $SU(2)_L$ anomaly by constructing three linearly independent currents(from equations (8) and (9)) not containing j^1_μ. The unbroken current is

$$\mathbf{J}^0_\mu = J^1_\mu - 2J^2_\mu \tag{10}$$

The remaining two (linearly independent) spontaneously broken global $U(1)$ currents are

$$\mathbf{J}^1_\mu = J^3_\mu - J^1_\mu$$
$$\mathbf{J}^2_\mu = \frac{35}{18} J^1_\mu - J^4_\mu \tag{11}$$

In terms of the first generation ordinary fermions, the two spontaneously broken global currents are

$$\mathbf{J}^1_\mu = -28(\overline{q^1_L}\gamma_\mu q^1_L + \overline{l^1_L}\gamma_\mu l^1_L - \overline{u^c_R}\gamma_\mu u^c_R - \overline{d^c_R}\gamma_\mu d^c_R - \overline{e^c_R}\gamma_\mu e^c_R) + \cdots$$

$$\begin{aligned}\mathbf{J}^2_\mu = & -4\overline{u^c_R}\gamma_\mu u^c_R - 2(\overline{d^c_R}\gamma_\mu d^c_R + \overline{e^c_R}\gamma_\mu e^c_R) \\ & - \frac{424}{9}(\overline{q^1_L}\gamma_\mu q^1_L + \overline{l^1_L}\gamma_\mu l^1_L - \overline{u^c_R}\gamma_\mu u^c_R - \overline{d^c_R}\gamma_\mu d^c_R - \overline{e^c_R}\gamma_\mu e^c_R) + \cdots\end{aligned} \tag{12}$$

There is no flavor mixing in this model, the CKM angles are assumed to arise from higher dimensional operators.

Our analysis has demonstrated that there are two potentially massless Goldstone bosons in the model. They couple with strength $2m_f/F_\pi$ where $F_\pi = 125$GeV to light fermions of current algebraic mass m_f and are experimentally ruled out [15].

Let us assume for the moment that these chiral symmetries are explicitly broken by physics beyond 1000 TeV. It is possible to obtain a rough estimate of the axion mass (m_A) from four fermion operators generated by heavy physics. Dashen's formula coupled with vacuum insertion approximation yields,

$$F_\pi^2 m_A^2 \approx \frac{\langle \overline{T}T\rangle^2_{\Lambda_S}}{F_S^2} \tag{13}$$

where $g_S F_S = \Lambda_S$ is the scale of new physics. Here $\langle \overline{T}T\rangle_{\Lambda_3} = 4 \times 10^8 \mathrm{GeV}^3$ is a rough estimate of the technifermion condensates used by Appelquist and Terning. Quoting values of the relevant anomalous dimensions from [12], we obtain

$$\langle \overline{T}T\rangle_{\Lambda_S} \approx < \overline{T}T >_{\Lambda_3} \left(\frac{100\,\mathrm{TeV}}{10\,\mathrm{TeV}}\right)^{0.67} \left(\frac{1000\,\mathrm{TeV}}{100\,\mathrm{TeV}}\right)^{0.32} \left(\frac{\Lambda_S \mathrm{TeV}}{1000\,\mathrm{TeV}}\right)^{\gamma_S} \tag{14}$$

where γ_S is the anomalous dimension from 1000 TeV to Λ_S. Hence

$$m_A \approx 32\left(\frac{10^3\mathrm{TeV}}{\mathrm{F_S}}\right)^{(1-\gamma_S)} \mathrm{GeV} \tag{15}$$

Here g_S is taken to be $O(1)$ and γ_S — a crude estimate of walking effects between Λ_S and Λ_{PS} —is also expected to be of $O(1)$.

In a $p\bar{p}$ collider, these neutral 'axions'($A_{1,2}$) may be produced singly via gluon fusion (predominantly) or by quark-antiquark fusion [16]. Note that there is not enough phase space for it to be produced in the pair technipion production mode ($W^{\pm} \rightarrow A_i P^{\pm}$), where $P^{\pm}$ is the charged pseudo-goldstone boson in the one-family technicolor model orthogonal to the 'eaten' goldstone bosons, as the mass of the $P^{\pm}$ is expected to be in excess of 50 GeV [4]. These axions would decay into fermion-antifermion pair, principally the heavier ones ($b\bar{b}$ and $\tau\bar{\tau}$). However, these decays are not of any experimental significance due to the smallness of $\frac{m_{b,\tau}}{F_\pi}$. Hence, a 32 GeV neutral, color-singlet 'axion' is unlikely to be detected in the near future.

A natural approach for creating these four-fermion operators would be to embed the Appelquist-Terning gauge group into a larger gauge group. This will break some of the chiral symmetries so that the axions are no longer strictly massless. The simplest possible extension (minimum fermion content and maximum chiral symmetry breaking) is to the gauge group $SU(9) \otimes SU(2)_{HC} \otimes SU(2)_L \otimes U(1)_R$; i.e., unification of the Pati-Salam and ETC gauge groups. The $SU(9)$ is assumed to break into $SU(5)_{ETC} \otimes SU(4)_{PS}$. The minimal representation content yielding us the eight representations in the Appelquist - Terning model , under decomposition, is

$$
\begin{aligned}
\Psi_1 &= (36,1,2)_0 = (10,1,1,2)_0 \oplus (5,1,4,2)_0 \oplus (1,1,6,2)_0 \\
\Psi_2 &= (36,1,1)_0 = (10,1,1,1)_0 \oplus (5,1,4,1)_0 \oplus (1,1,6,1)_0 \\
\Psi_3 &= (\overline{36},2,1)_0 = (\overline{10},2,1,1)_0 \oplus (\overline{5},2,\overline{4},1)_0 \oplus (1,2,6,1)_0 \\
\Psi_4 &= (\overline{36},1,1)_{-1} = (\overline{1}0,1,1,1)_{-1} \oplus (\overline{5},1,\overline{4},1)_{-1} \oplus (1,1,6,1)_{-1} \\
\Psi_5 &= (\overline{36},1,1)_1 = (\overline{1}0,1,1,1)_1 \oplus (\overline{5},1,\overline{4},1)_1 \oplus (1,1,6,1)_1 \qquad (16) \\
\Psi_6 &= (9,1,1,1)_0 = (5,1,1,1)_0 \oplus (4,1,1,1)_0 \\
\Psi_7 &= (\overline{9},1,1)_0 = (\overline{5},1,1,1)_0 \oplus (\overline{4},1,1,1)_0 \\
\Psi_8 &= (126,1,1)_0 \\
&= (\overline{5},1,1,1)_0 \oplus (1,1,1,1)_0 \oplus (10,1,6,1)_0 \oplus (\overline{1}0,1,4,1)_0 \\
&\quad \oplus (5,1,\overline{4},1)_0
\end{aligned}
$$

Since we want to avoid Non-Abelian Goldstone bosons, no representations participating in any condensate should be repeated. The original fermions are contained in Ψ_1, ...,Ψ_6. The representations Ψ_7 and Ψ_8 are needed to cancel gauge anomalies.

However, this is not a resolution. While it explicitly breaks some chiral symmetries, the additional representations create new chiral symmetries. The 'axions' are still there.

Another approach for tackling this problem would be to gauge the two $U(1)$'s that are spontaneously broken at the electroweak scale.(However, this would require a new set of spectator fermions so that the theory is anomaly free. One needs to assume that they are singlets under all but these two $U(1)$ groups and get a mass from heavy physics.) Then, the two massless $U(1)$ gauge bosons combine with the two massless goldstone bosons to give the former their masses. The arbitrariness of the $U(1)$ gauge coupling can push the gauge boson masses beyond their current experimental limits. $Z^{0'}$ mass (lower)bounds

indicate that the U(1) gauge couplings would have to be rather large to be compatible with experiment.

Another possibility would be to arrange to have more than one condensate at higher energy gauge symmetry breaking scales. As a result, the $U(1)$'s are spontaneously broken at higher energies and hence are more weakly coupled to the light fermions (weak enough to be undetected). This way one ensures that there are no light goldstone bosons at the electroweak scale. However, 'f_π' would have to be between 10^{10} GeV and 10^{12} GeV from cosmological [17] and astrophysical considerations [18].

The above discussion can be stated in more general terms. A reason for the problem is well known—too many representations [6]. If the number of irreducible representations (n_D) is less than or equal to the number of simple non-abelian gauge group factors (n_S) there will be no anomaly-free global U(1) currents to worry about.

If $n_D > n_S$, we have $(n_D - n_S)$ anomaly-free global U(1) symmetries. We then investigate the fate of these symmetries as we pass through the various gauge symmetry breaking scales. Suppose the gauge symmetry breaking at Λ involves c condensates and there are d broken diagonal generators which act as the number operator in the subspace of the fermions in the condensate. (For instance, when $SU(n)$ breaks into $SU(n-1)$, the element of the Cartan subalgebra in $SU(n)$ but not in $SU(n-1)$ acts as the number operator in the 1 and $n-1$ subspace separately.) Consider the general linear combination of these $(n_D - n_S + d)$ $U(1)$ currents ($J = \sum A_r J^r_\mu$, $r = 1, \cdots (n_D - n_S + d)$) and tabulate the charges Q_i, $(i = 1, \cdots c)$ of the c condensates. (Without loss in generality, we are assuming no global symmetry breaking above this scale.) Note that these Q_i are linear combinations of $(n_D - n_S + d)$ variables A_r.

The conditions that the charges of the c condensates vanish can be stated as a set of c linear homogeneous equations in $(n_D - n_S + d)$ variables. If the rank of the coeficient matrix (c') is less than c, only c' condensates are said to have linearly independent charges. We focus on the c' linearly independent condensates i.e., those condensates with linearly independent charges. Only when $c' = d$ are the initial $U(1)^{(n_D - n_G)}$ global symmetries preserved; otherwise there are $c' - d$ exactly massless Goldstone bosons, which would be unwelcome if they couple to the ordinary fermions. Gauging these $U(1)$'s is a solution, but this entails the introduction of spectator fermions.

We know of no simple way of determining c' other than to explicitly construct the global symmetry currents at each stage of the gauge symmetry breaking. In any case, the analysis needs to be carried out only when $c > d$; i.e., when the number of condensates is

larger than the number of 'diagonal' gauge generators spontaneously broken at that scale. In particular, there could be many (unrepeated) ETC representations as long as $c \leq d$.

4. CONCLUSION

We have shown the existence of two potentially light Goldstone bosons in the Appelquist-Terning ETC model. The problem is related to the fact that there are too many representations and the number of condensates exceeds the number of diagonal gauge symmetries broken. These axions could get a mass from physics at higher energy scales. An attempt to break these symmetries with new gauge interactions seems to be inadequate.

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References

[1] K. G. Wilson, *Phys. Rev.* **D3**, 181 (1971)

[2] R. Dashen and H. Neuberger, *Phys. Rev. Lett.* **50** (1983) 1897

[3] S. Weinberg, *Phys. Rev.* **D19** (1979) 1277; L. Susskind, *Phys. Rev.* **D20** (1979) 2619

[4] B. Holdom, *Phys. Rev.* **D24** (1981) 1441; *Phys. Lett.* **150B** (1985) 301; T. Appelquist, D. Karabali and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **57** (1986) 957; T. Appelquist and L. C. R. Wijewardhana, *Phys. Rev.* **D36** (1987) 578; K. Yamawaki, M. Bando and K. Matumoto, *Phys. Rev. Lett.* **56**, (1986) 1335; T. Akiba and T. Yanagida,*Phys. Lett.* **169B** (1986) 432

[5] S. Dimopoulos and L. Susskind, *Nucl. Phys.* **B155** 237

[6] E. Eichten, K. Lane, *Phys. Lett.* **B90** 125

[7] D. C. Kennedy and B. W. Lynn, *Nucl. Phys.* **B322**, 1 (1989)

[8] M. Golden and L. Randall, *Nucl. Phys.* **B361**,3 (1991); B. Holdom and J. Terning, *Phys. Lett.* **B247**, 88 (1990); M. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990); A. Dobado, D. Espriu, and M. Herrero, *Phys. Lett* **B253**, 161 (1991); M. Peskin and T. Takeuchi, *Phys. Rev.* **D46** (1992) 381; R. S. Chivukula, M. Dugan and M. Golden, *Phys. Lett.* **B292** (1992) 435

[9] M. Einhorn, D. Jones, and M. Veltman, *Nucl. Phys.* **B191** (1981) 146; A. Cohen, H. Georgi, and B. Grinstein *Nucl. Phys.* **B232** (1984) 61

[10] R.Sundrum and Stephen D. H. Hsu, *Nucl. Phys.* **B391** (1993) 127; Markus A. Luty and R. Sundrum, *Phys. Rev. Lett.* **70** (1993) 529; T. Appelquist and J. Terning, *Phys. Lett.* **B315** (1993) 139

[11] R. S. Chivukula, S. B. Selipsky, and E. H. Simmons, *Phys. Rev. Lett.* **69** (1992) 575; R. S. Chivukula, E. Gates, E. H. Simmons, and J. Terning, *Phys. Lett.* **B311** (1993) 157

[12] T. Appelquist and J. Terning, **YCTP-P21-93 BUHEP-93-23**, hep-ph/9311320

[13] S. Dimopoulos, S. Raby, and L. Susskind, *Nucl. Phys.* **B169** (1980)373

[14] K. Lane, *Proceedings of the The Fifth Johns Hopkins Workshop on Current Problems in Particle Theory, Baltimore, Maryland* 1981

[15] T. W. Donnelly *et. al.*, *Phys. Rev.* **D18**, (1978) 1607; S. Barshay *et. al.*, *Phys. Rev. Lett.* **46**, (1981); A. Barroso *et. al.*, Phys. Lett. **106B**, (1981) 91; and R.D. Peccei, in *Proceedings of Neutrino'81*, Honololu, Hawaii, Vol.1, p.149 (1981)

[16] E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, *Rev. Mod. Phys.* **56** (1984) 579

[17] J. Preskill *et. al.*, *Phys. Lett.* **120B**, (1983) 127; L. F. Abbott, and P. Sikivie, *Phys. Lett.* **120B**, (1983) 133; and M. Dine and W. Fischler,*Phys. Lett.* **120B** (1983) 137

[18] D. A. Dicus *et al.*, *Phys. Rev.* **D18** (1978); M. Fukugita, S. Watamura, and M. Yoshimura, *Phys. Rev. Lett.* **48**, (1982) 1522, also *Phys. Rev.* **D26**, (1982) 1840; D. S. P. Dearborn *et al.*, Phys. Rev. Lett. **56** (1986) 26; and G. G. Raffelt, *Phys. Rev.* **D33**, (1986) 897